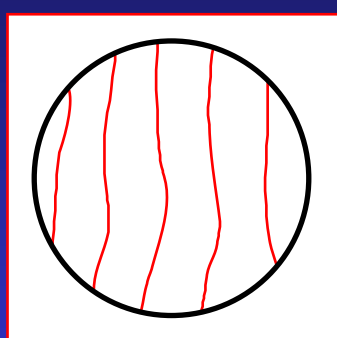
Part 2
Phase-Shifting Interferometry

- Elementary analysis of interferograms
- Basic algorithms
- Removing phase ambiguities
- Single shot phase shifting (Reducing effects of vibration)

Typical Interferogram

Surface Error = $(\lambda/2)\ (\Delta/S)$

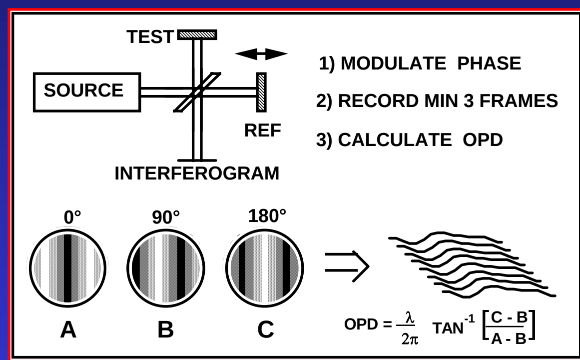


Classical Analysis

Measure positions of fringe centers.

Deviations from straightness and equal spacing gives aberration.

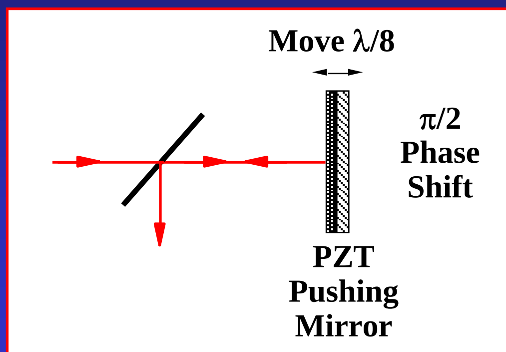
Phase-Shifting Interferometry



Advantages of Phase-Shifting Interferometry

- High measurement accuracy (>1/1000 fringe, fringe following only 1/10 fringe)
- Rapid measurement
- Good results with low contrast fringes
- Results independent of intensity variations across pupil
- Phase obtained at fixed grid of points
- Easy to use with large solid-state detector arrays

Phase Shifting - Moving Mirror



Four Step Method

phase shift

$$I(x,y) = I_{dc} + I_{ac}\cos[\phi(x,y) + \phi(t)]$$

measured object phase

$I_1(x,y) = I_{dc} + I_{ac}\cos[\phi(x,y)]$ $\quad \phi(t) = 0 \quad (0°)$

$I_2(x,y) = I_{dc} - I_{ac}\sin[\phi(x,y)]$ $\quad = \pi/2 \quad (90°)$

$I_3(x,y) = I_{dc} - I_{ac}\cos[\phi(x,y)]$ $\quad = \pi \quad (180°)$

$I_4(x,y) = I_{dc} + I_{ac}\sin[\phi(x,y)]$ $\quad = 3\pi/2 \quad (270°)$

$$\mathrm{Tan}[\phi(x,y)] = \frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}$$

Relationship between Phase and Height

$$\phi(x,y) = Tan^{-1}\left[\frac{I_4(x,y) - I_2(x,y)}{I_1(x,y) - I_3(x,y)}\right]$$

$$\text{Height Error}(x,y) = \frac{\lambda}{4\pi}\phi(x,y)$$

Phase-Measurement Algorithms

Three Measurements

$$\phi = \tan^{-1}\left[\frac{I_3 - I_2}{I_1 - I_2}\right]$$

Four Measurements

$$\phi = \tan^{-1}\left[\frac{I_4 - I_2}{I_1 - I_3}\right]$$

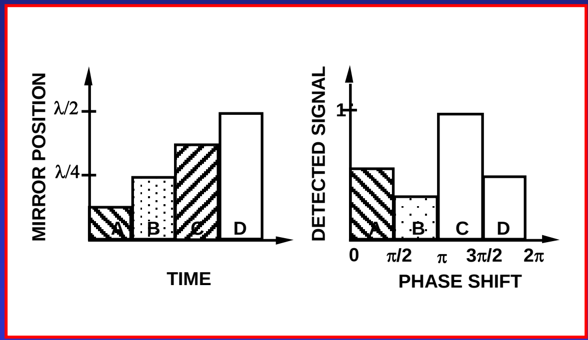
Schwider-Hariharan Five Measurements

$$\phi = \tan^{-1}\left[\frac{2(I_2 - I_4)}{2I_3 - I_5 - I_1}\right]$$

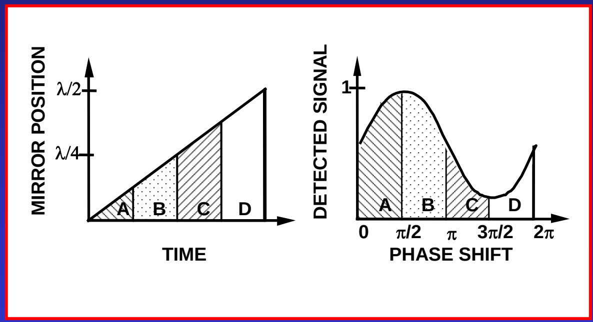
Carré Equation

$$\phi = \tan^{-1}\left[\frac{\sqrt{[3(I_2 - I_3) - (I_1 - I_4)][(I_2 - I_3) - (I_1 - I_4)]}}{(I_2 + I_3) - (I_1 + I_4)}\right]$$

Phase-Stepping Phase Measurement



Integrated-Bucket Phase Measurement



Integrating-Bucket and Phase-Stepping Interferometry

Measured irradiance given by

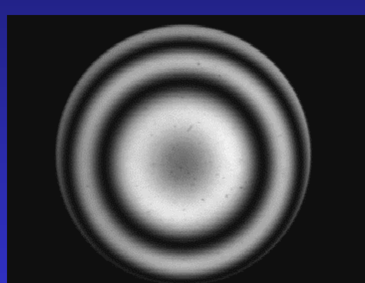
$$I_i = \frac{1}{\Delta}\int_{\alpha_i-\Delta/2}^{\alpha_i+\Delta/2} I_o\{1+\gamma_o\cos[\phi+\alpha_i(t)]\}\,d\alpha(t)$$

$$= I_o\left\{1+\gamma_o \mathrm{sinc}\left[\frac{\Delta}{2}\right]\cos[\phi+\alpha_i]\right\}$$

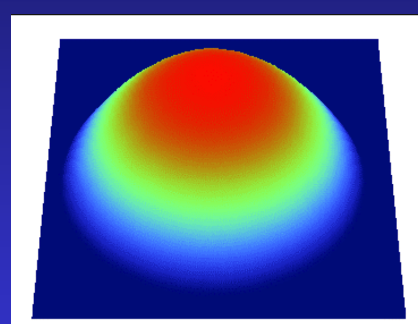
Integrating-Bucket $\Delta=\alpha$

Phase-Stepping $\Delta=0$

Typical Fringes For Spherical Surfaces



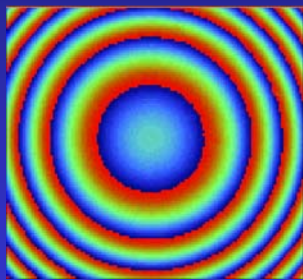
Fringes

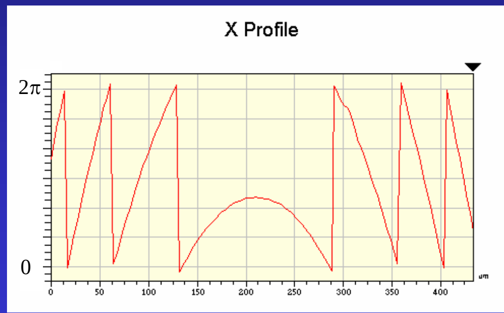


Phase map

Phase Ambiguities
-Before Unwrapping

2 π Phase Steps





Removing Phase Ambiguities

- Arctan Mod 2π (Mod 1 wave)
- Require adjacent pixels less than π difference (1/2 wave OPD)

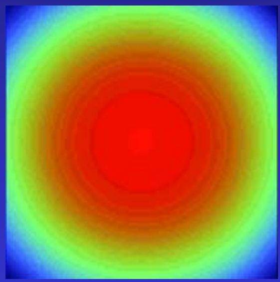
- Trace path
- When phase jumps by > π

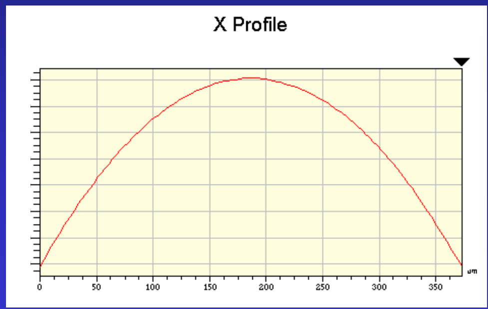
  Add or subtract N2π

  Adjust so < π

Phase Ambiguities - After Unwrapping

Phase Steps Removed



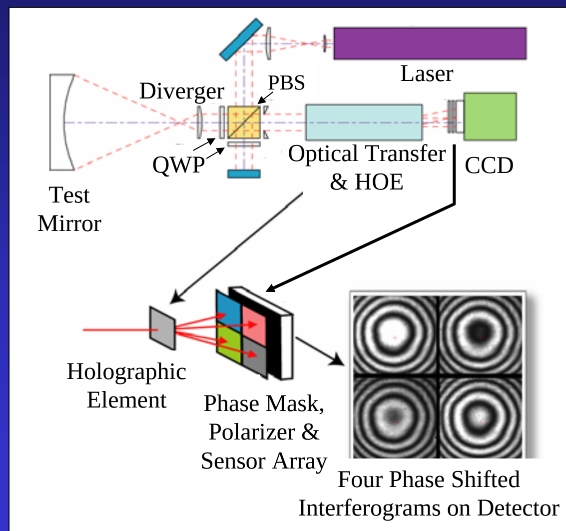


Error Due to Vibration

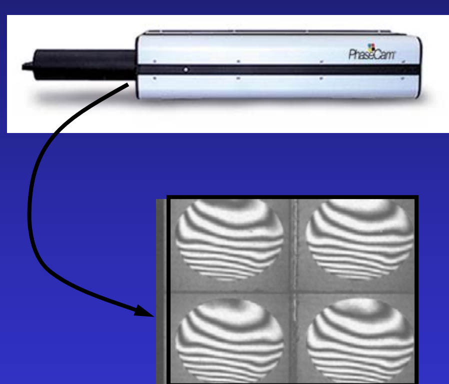
- Probably the most serious impediment to wider use of PSI is its sensitivity to external vibrations.
- Vibrations cause incorrect phase shifts between data frames.
- Error depends upon frequency of vibration present as well as phase of vibration relative to the phase shifting.

4D PhaseCam Operation

- **Twyman-Green**
  - **Two beams have orthogonal polarization**
- **4 Images formed**
  - **Holographic element**
- **Single Camera**
  - **1024 x 1024**
  - **2048 x 2048**
- **Polarization used to produce 90-deg phase shifts**



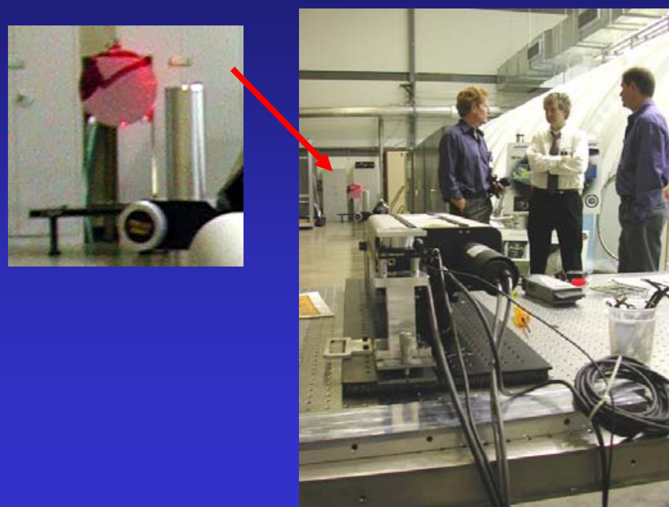
Effects of vibration



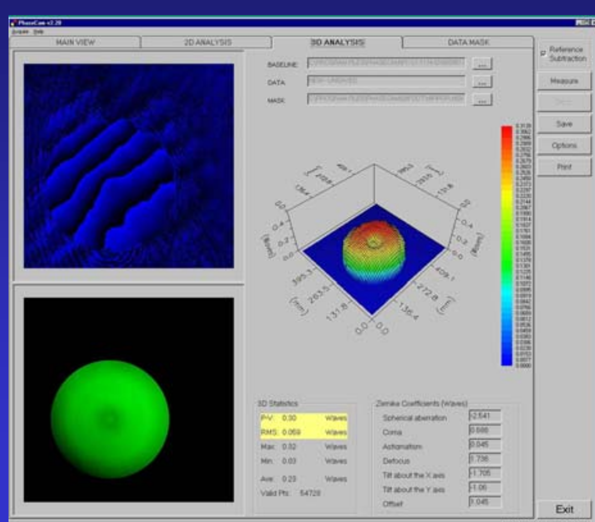
Relative motion between PhaseCam and test object

Phase relationship is fixed

Testing a 0.5 meter diameter, 20 meter ROC mirror



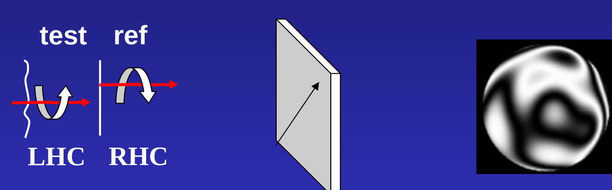
0.5 m diameter mirror, 20 m ROC 5 nm rms repeatability (in air)



Pixelated Phase Sensor

- **Compacted pixelated array placed in front of detector**
- **Single frame acquisition**
  - **High speed and high throughput**
- **Achromatic**
  - **Works from blue to NIR**
- **True Common Path**
  - **Can be used with white light**

Use polarizer as phase shifter

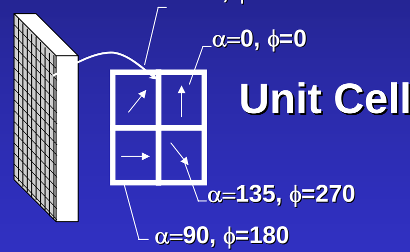


Circ. Pol. Beams ($\Delta\phi$) + linear polarizer (α) ⟶ cos ($\Delta\phi + 2\alpha$)

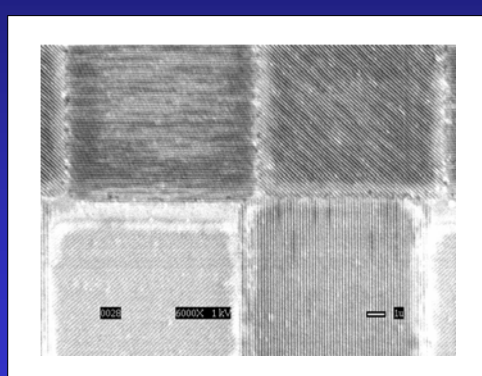
Phase-shift depends on polarizer angle

Array of oriented micropolarizers

Polarizer array
Matched to
detector array
pixels

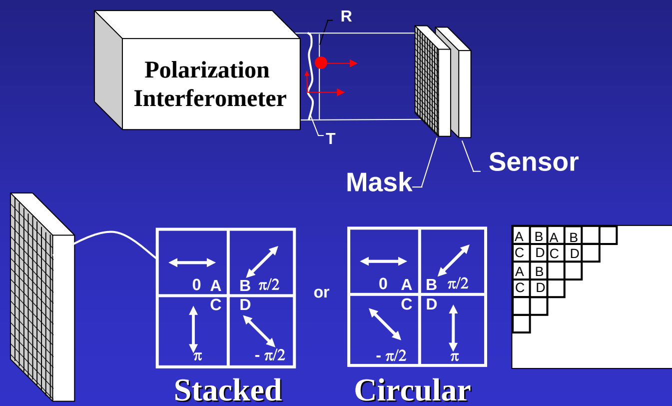


Electron micrograph of wire grid polarizers

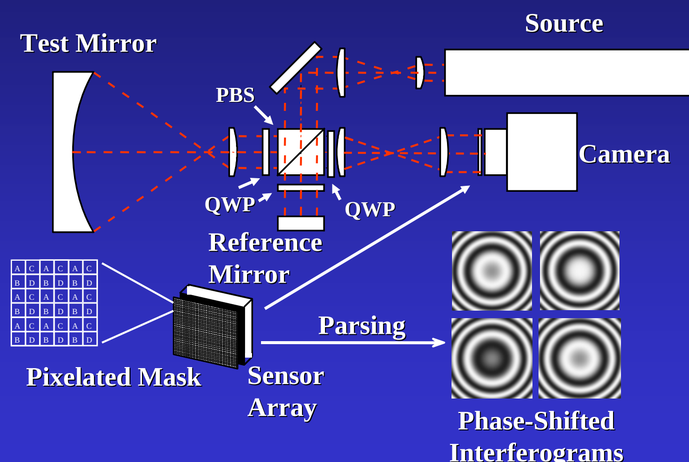


20 um

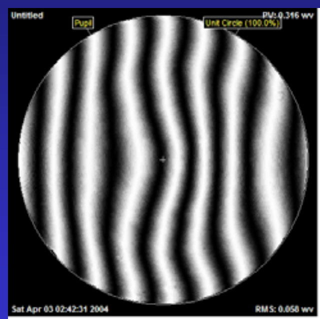
Array of phase-shift elements unique to each pixel

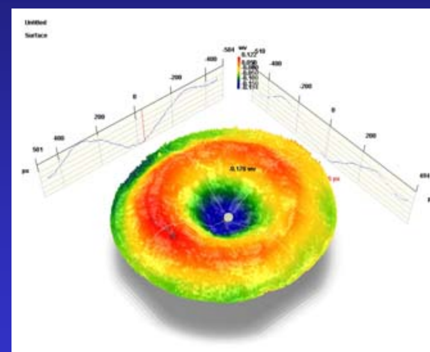


System Configuration



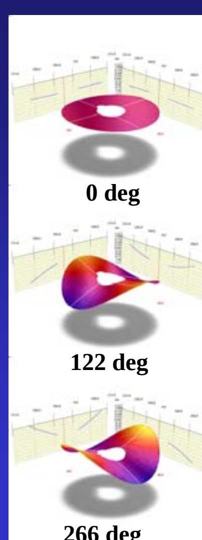
Measurement of 300 mm diameter, 2 meter ROC mirror

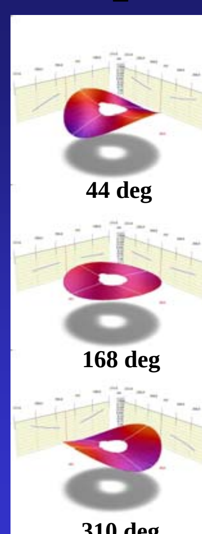


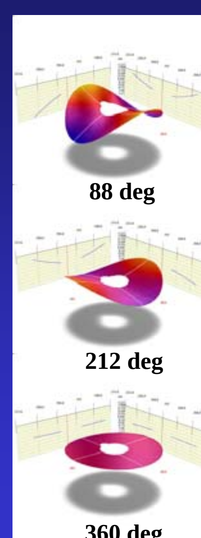


Mirror and interferometer on separate tables!

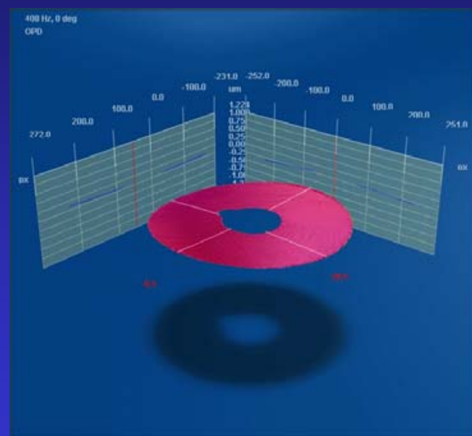
Phase Sweep at 408 Hz





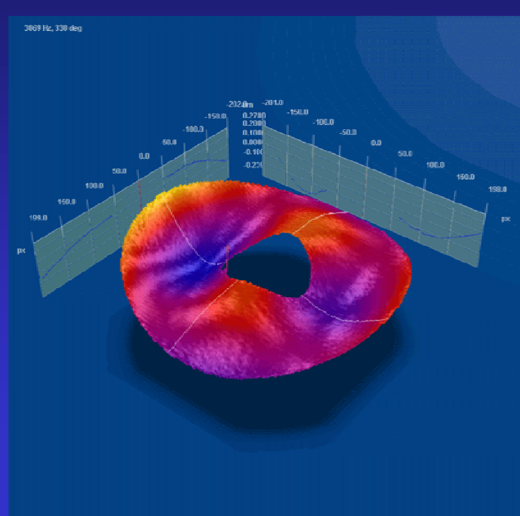


Modal Movie - Disk Vibration

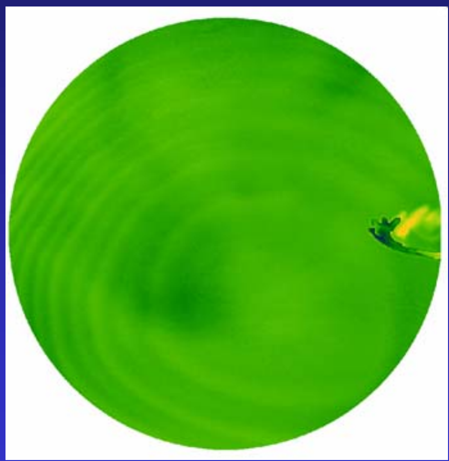


Al mirror, 408 Hz

Hard Disk Platter Excited by PZT at 3069 Hz



Measurement Results



Air burst
Phase w/reference subtraction

Conclusions – Single Shot Interferometer

- Vibration insensitive, quantitative interferometer
- Surface figure measurement (nm resolution)
- Snap shot of surface height
- Acquisition of "phase movies"